Scaffolding the key ideas and strategies in Number

A Professional Development Day

presented by

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RMIT University
OVERVIEW:

• Lessons from research
• Early Number ideas and strategies
• Developing place-value
• Extending Place-value
• Second Place-Value Pattern
LESSONS FROM RESEARCH

Research on teaching and learning and developments in our technological society have prompted considerable changes in how mathematics is taught.

School mathematics NOW involves interaction and negotiation of the ‘big ideas’. Contemporary approaches include: extended investigations, rich tasks, open-ended questions, games, discussion of solution strategies, mental computation, and visualisation.
It is now recognised that teachers not only need to know the **key concepts, skills and strategies** that underpin primary mathematics (**content knowledge**), teachers also need a **deep knowledge of the links** between these ideas, what makes them difficult, and how they are best taught and learnt (**pedagogical content knowledge**).  

**Teachers remain the single most important influence on childrens’ mathematics learning** (Hattie, 2003)
We also know a lot more about how children learn mathematics.

Meaningless rote-learning, mind-numbing, text-based drill and practice, and doing it one way, the teacher’s way, does not work.

Concepts need to be experienced, strategies need to be scaffolded and EVERYTHING needs to be discussed to learn with understanding.
A NEW FOCUS

One of the main aims of school mathematics is to create mental objects in the mind’s eye of children which can be manipulated flexibly with understanding and confidence.

A prolonged reliance on inefficient strategies such as ‘make-all-count-all’ is both developmentally dangerous and professionally irresponsible.

Dianne Siemon, 2000
Two Aspects:

NUMERATION - An understanding of number concepts and notation, specifically those understandings and skills needed to model, name, write, read, interpret, and use:

- Natural Numbers \([1, 2, 3, 4, \ldots]\)
- Whole numbers \([0, 1, 2, 3, \ldots]\)
- Integers \([-2, -1, 0, 1, 2, 3, \ldots]\)
- Fractions \([m/n: \text{where } m \& n \text{ are integers, } n \neq 0]\)
OPERATIONS - An understanding of the concepts, strategies and skills needed to support computation and estimation for whole numbers, decimal fractions, common fractions and per cent

<table>
<thead>
<tr>
<th></th>
<th>Materials/Models</th>
<th>Mental Computation</th>
<th>Estimation</th>
<th>Written Calculations</th>
<th>Technology</th>
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<td>Multiplication</td>
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<td>Division</td>
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THE NUMBERS 0 – 9:

Materials
Real-world, stories

Language
read, say, write

Symbols
recognise, read, write

Make

Perceptual Learning

Name
five

Record

No Distractors

Distractors
COUNTING: “Jenni can count to 100 ...”

To count effectively, children not only need to know the number naming sequence, they need to recognise that:

- counting objects and words need to be in one-to-one correspondence;
- “three” means a collection of three no matter what it looks like;
- the last number counted tells ‘how many’.

“But can Jenni read numbers without counting?”

To develop a strong sense of number, children also need to be able to:

• recognise collections up to five without counting (*subitising*); and

• name numbers in terms of their parts (*part-part-whole relationships*).

Eg, recognise “3” instantly and see this collection as a “2 and a 1 more”
How many?
What did you see?
Try this:
What did you see?
Or this:
What did you see?
But what about?
The numbers 0 to 9 are the only numbers most of us ever need to learn ... it is important to know everything there is to know about each number. For this collection, we need to know that it:

- can be counted by matching number names to objects: “one, two, three, four, five, six, seven, eight” and that the last one says, how many;
- can be described as eight or 8; and it
- is 1 more than 7 and 1 less than 9.
But we also need to know 8 in terms of its parts and how it relates to 10 – this is called **part-part-whole** knowledge, that is,

- 8 is 2 less than 10
- 6 and 2 more
- 4 and 4
- Double 4
- 3 and 3 and 2
- 5 and 3, 3 and 5
- 2 less than 10

Differently configured ten-frames are ideal for this.
• Interpret/visualise numbers beyond ten:

8 and 8 … 16
1 ten and 6 more … 16

To build a sense of numbers beyond ten
DEVELOP early number and part-part-whole ideas by providing regular opportunities to make, name and record numbers to ten, count and compare collections, and subitise:

- Materials
- Number Cards
- Ten-frames
- Dice, Dominoes
- Part-whole Cards
- Games

Fold-over flaps
TRUSTING THE COUNT:
This recently recognised capacity* builds on these early number ideas.

* See WA Department of Education, First Steps in Mathematics

**Trusting the count** has a range of meanings:

- Initially, children may not believe that if they counted the same collection again, they would get the same result, or that counting is a strategy to determine how many.

- Ultimately, it is about having access to a range of mental objects for each of the numerals, 0 to 9, which can be used flexibly without having to make, count or see these collections physically.

* See WA Department of Education, *First Steps in Mathematics*
Trusting the count is evident when children:

- know that counting is an **appropriate response** to questions which ask how many;
- believe that counting the same collection again will always produce the **same result** irrespective of how the objects in the collection are changed or manipulated;
- are able to invoke a range of **mental objects** for each of the numbers 0 to ten (including part-part-whole knowledge, visual imagery);
- **work flexibly with numbers** 0 to ten using part-part-whole knowledge and/or visual imagery without having to make or count the numbers; and
- are able to use small collections as units when counting larger collections.
Counting Strategies:

- **Make all – count all** (physical models, one-to-one correspondence)
- **Count on from covered** (physical models and numerals, count on from hidden part)
- **Count on/back from larger** (physical models and numerals, count on/back from larger number)
- **Skip counting** (physically count by twos, then fives and tens) to determine how many
DEVELOPING PLACE-VALUE:

Children can be formally introduced to place-value as a system of recording numbers when they:

- can **comfortably count** to 20 and beyond;
- are well-acquainted with the numbers 0 to 10 in terms of their parts (**part-part-whole relations**);
- can work flexibly with numerals to 10 without having to model the count (**trust the count**);
- interpret/visualise numbers beyond ten in terms of 1 ten and 4 more, “fourteen”;
- recognise numbers to 10 as **countable units** for the purposes of counting, eg, 2, 4, 6, 8 ...
Inconsistencies in the English number naming sequence for 2-digit numbers:

<table>
<thead>
<tr>
<th>English</th>
<th>Vietnamese</th>
<th>English</th>
<th>Vietnamese</th>
</tr>
</thead>
<tbody>
<tr>
<td>mot</td>
<td>1</td>
<td>muoi mot</td>
<td>11</td>
</tr>
<tr>
<td>hai</td>
<td>2</td>
<td>muoi hai</td>
<td>12</td>
</tr>
<tr>
<td>ba</td>
<td>3</td>
<td>muoi ba</td>
<td>13</td>
</tr>
<tr>
<td>bon</td>
<td>4</td>
<td>muoi bon</td>
<td>14</td>
</tr>
<tr>
<td>nam</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sau</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bay</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tam</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>chin</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>muoi</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Vietnamese Number Naming Sequence

NB: consistency for teens and beyond

<table>
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<tr>
<td>hai</td>
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<td>muoi</td>
</tr>
<tr>
<td>tam</td>
<td>muoi chin</td>
</tr>
</tbody>
</table>

The sequence for 10-20 is consistent with the remaining digits.
INTRODUCING PLACE-VALUE:

1. Establish the **new unit** – 10 ones is 1 ten
2. Introduce the **names** for the multiples of ten.
3. **Make, name** and **record regular** examples of the 2-digit place-value pattern
4. **Make, name** and **record the teen numbers**.
5. **Consolidate** through comparing, ordering, counting forwards and backwards in place-value parts and renaming.

Place-value is all about pattern recognition and use – it is essentially multiplicative
1. Establish the new unit: **10 ones is 1 ten**

Make and count tens using bundling materials (icy-pole sticks, straws) or connectables (unifix)

Treat tens as countable units

Why is “units” appropriate here?
2. Introduce names for multiples of ten

   Establish regular names before ‘irregular’
   names, emphasise pattern

   Eg, six-ty, seven-ty, eight-y, nine-ty

   (cardinal)

   thirty (should be three-ty) fifty (should be five-ty)

   (ordinal)

   twenty (should be two-ty), forty (should be four-ty)

   (mispelt)

   There is no one-ty to support the pattern
3. Make, name & record tens and ones: for 20 – 99 (regular numbers first)

<table>
<thead>
<tr>
<th>“Make me ...”</th>
<th>read, write, name</th>
<th>record</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 tens 7 ones</td>
<td>sixty-seven</td>
<td>6 7</td>
</tr>
<tr>
<td>3 tens 4 ones</td>
<td>thirty-four</td>
<td>3 4</td>
</tr>
<tr>
<td>4 tens 0 ones</td>
<td>forty</td>
<td>4 0</td>
</tr>
</tbody>
</table>

3 of these and 4 of those
4. Make, name & record tens and ones: for 10 – 19 (least irregular first)

<table>
<thead>
<tr>
<th>“Make me ...”</th>
<th>read, write, name</th>
<th>record</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ten 8 ones</td>
<td>eighteen</td>
<td>1 8</td>
</tr>
<tr>
<td>1 ten 5 ones</td>
<td>fifteen</td>
<td>1 5</td>
</tr>
<tr>
<td>1 ten 2 ones</td>
<td>twelve</td>
<td>1 2</td>
</tr>
</tbody>
</table>

Using a variety of bundling materials
CONSOLIDATING PLACE-VALUE

• **Compare** 2 numbers using multiple representations (materials, words, symbols), say which is larger/smaller and why

• **Order (sequence)** more than 2 numbers from smallest to largest, give reasons why (eg, place number cards on a 0 to 100 rope)

• **Count forwards and backwards in place-value parts** starting from anywhere

• **Rename numbers** in more than one way

Continue to **make, name and record**
Eg, A Place-Value Game
If this shape had to cover 24, what is the largest number it could cover?
Eg, Apply what is known

**Tallies:** eg, dice sums

Highest sum? Lowest sum? Keep a record? What do you notice?

**Graphs and Charts:** eg, birthdays, eye-colour

**Money:** eg, I have $18 in my pocket. What notes and coins might I have?

**Measurement:** eg, Find something that is longer than 53 cm but shorter than 94 cm
EXTENDING PLACE-VALUE:

1. Introduce the new unit – 10 tens is 1 hundred using Multi-base Arithmetic Blocks (MAB)

2. Make, name and record - regular examples such as 486 and 178 before more difficult examples such as 417, 713, 205 and 700

3. Consolidate through making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming

Examples?
Eg, Developing 4-digit numeration:

1. **Establish the new unit**: 10 hundreds is 1 thousand

   Make and count thousands using Multi-base Arithmetic Blocks (MAB)

   ![1 thousand](image1) ![2 thousands](image2) ![3 thousands](image3) ![4 thousands](image4)

   1 thousand 2 thousands 3 thousands 4 thousands ...

2. Make, name & record thousands, hundreds, tens and ones

“Make me 4 thousands, 3 hundreds, 7 tens and 6 ones”

It’s said and read as: “4 thousand 3 hundred and seventy-six”

Record:

<table>
<thead>
<tr>
<th></th>
<th>thousands</th>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>6</td>
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<tr>
<td></td>
<td>7</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Take special care with internal zeros and teens
3. Consolidate

- **Compare** 2 numbers using multiple representations (materials, words, symbols), say which is larger/smaller and why

- **Order (sequence)** more than 2 numbers from smallest to largest, give reasons why

- **Count forwards and backwards in place-value parts** starting from anywhere

- **Rename numbers** in more than one way – read to place-value parts to say how many.

Use ‘real-world’ examples wherever possible
Eg, Extended Number Chart Activity

Complete and describe the counting pattern.
Eg, Another Place-Value Game

Draw this arrangement of boxes

Take it in turns to throw a single dice. Use the numbers to make a 1-digit, 2-digit and 3-digit number. Winner is person with highest sum.
THE SECOND PLACE-VALUE PATTERN:

As the number of place-value parts increases it becomes too cumbersome to name every part - A more efficient naming system is needed.

This system, referred to as the Second Place-Value Pattern, involves the repeated use of hundreds, tens and ones to count certain units (eg, thousands, millions, billions, trillions ...) from 1 to 999 instead of 1 to 9, eg,

“387 billion 562 million 408 thousand 571” (ones)
Eg, Number-Naming Dice Activity

Use a ten-sided dice and record from right to left:

| 2 | 6 | 7 | 4 | 0 | 5 | 8 |

Challenge: Use a ten-sided dice and record from left to right:

| 8 | 5 | 0 | 4 | 7 | 6 | 2 |
Eg, Developing 5-digit numbers:

1. Introduce the new unit, 10 thousands is 1 ten thousand using MAB

2. Introduce names for multiples of new unit
   10 thousand  20 thousand  30 thousand ....
   (NB: Should be 1 ten thousand, 2 ten thousand, ......)

3. Represent, name and record

   24,376
   
<table>
<thead>
<tr>
<th>ten thousands</th>
<th>thousands</th>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>7</td>
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</table>

4. Consolidate
   NB: Commas generally used to make it easier to read larger numbers
Consolidating 5-digit numbers:

1. Compare (compare car prices)
2. Sequence (order football attendances)
3. Count forwards and backwards
   
   34,569  34,658  34,747  ........?

4. Rename
   47092
   4 ten thousands 7 thousands 0 hundreds 9 tens 2 ones
   47 thousands 92 ones

Make a Number Expander
   4 ten thousands 709 tens 2 ones
   470 hundreds 9 tens 2 ones
Making a Number Expander:

<table>
<thead>
<tr>
<th>3</th>
<th>thousands</th>
<th>8</th>
<th>hundreds</th>
<th>6</th>
<th>tens</th>
<th>9</th>
<th>ones</th>
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</table>

Allow 3 rectangles per place-value part, 1 for the numeral and 2 for the place name.

Pinch and fold to enable numerals to sit side by side.
Rounding:

Rounding is a form of comparing – it is generally used to support estimation.

Read to the required place then look to the next place to determine closest value.

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Nearest ten? 45,672 Nearest hundred?

See Booker et al (2003) pp.120-124